REFRACTIVE-RAY BENDING IN AXIALLY-SYMMETRIC PLASMA SOURCES

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Abstract—It has been shown that the refraction of radiation in circularly symmetrical, optically-thin plasma sources may be the cause of systematic errors in spectroscopic measurements. The magnitude of this error depends upon the refractive index gradient and the plasma length.

A numerical procedure is described which permits evaluation of the light trajectory in an axisymmetric plasma.

INTRODUCTION

It is well known that, whenever a beam of light passes through a non-uniform medium, deviation must occur towards the region of higher refractive index.[1] This effect forms the basis of the Schlieren technique so frequently used for the investigation of plasmas.[2] The importance of the angular deviation of the light ray, when it passes through an inhomogenous plasma in interferometric measurements, has been studied in the past.[2-4] However, this same effect has been utterly neglected in the field of plasma spectroscopy.

The aim of this paper is to draw attention to possible sources of systematic error in the spectroscopic measurements caused by refraction of radiation in circularly symmetrical plasma sources.

THEORY

The angular deviation of a light ray in a non-homogenous medium may be estimated from the radius of curvature, R, of the deviated path and is given by[1]

$$(1/R) = (1/\mu)(\nabla\mu \cdot \mathbf{n}), \tag{1}$$

where μ is the local phase refractive index and $\mathbf{n}$ is the local unit vector perpendicular to the ray. Since this work deals with axial investigations of plasmas, only deflections of the ray for a cylindrically-symmetric geometry, i.e., the radial distribution of the refractive index, will be considered here in detail. For this two-dimensional model, the "x" axis coincides with the plasma axis and is assumed to be parallel to the incident ray. The differential equation of the ray path, obtained from equation (1), is therefore

$$\frac{\mathrm{d}^2y/\mathrm{d}x^2}{|1+(\mathrm{d}y/\mathrm{d}x)^2|^{3/2}} = \frac{1}{\mu}\frac{\mathrm{d}\mu}{\mathrm{d}y}\frac{1}{|1+(\mathrm{d}y/\mathrm{d}x)^2|^{1/2}}. \tag{2}$$

Integration of equation (2) gives

$$(\mathrm{d}y/\mathrm{d}x)_n = \sqrt{(\mu_n/\mu_0)^2 - 1}, \tag{3}$$

where μ_n is the local refractive index and μ_0 is the refractive index at the point of incidence of the ray. The radius of curvature of this trajectory is given by

$$R_n = \frac{\mu_n}{\mu_0(\mathrm{d}\mu/\mathrm{d}y)_n}. \tag{4}$$

The radial variation of the refractive index is given, approximately, by the following relation:

$$\mu_n = \mu_a - b\exp(-cy_n), \tag{5}$$

where μ_a is the refractive index at the plasma axis and b and c are constants. It is not, however, possible to solve equation (3) analytically. In order to obtain the ray trajectory, it is necessary to employ a numerical method.

NUMERICAL PROCEDURE

The ray trajectory is obtained by determining the equation of the corresponding circle and its subsequent evolution at the starting point of the ray trajectory. The next point of the trajectory is computed from this equation and the whole procedure is then repeated. This development may be easily understood from the following equations and the notation given in Fig. 1.

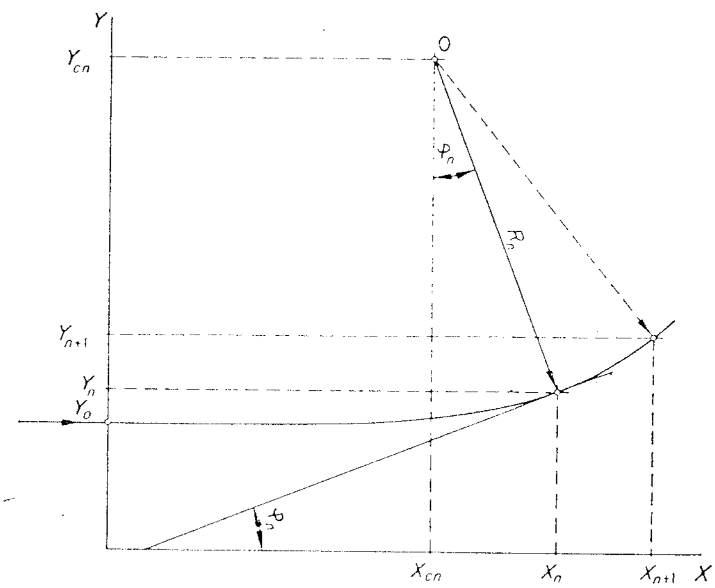

Fig. 1. Illustration of the numerical procedure.

Starting from the equation of the circle in the X–Y plane

$$(Y_{n+1} - Y_{cn})^2 + (X_{n+1} - X_{cn})^2 = R_n^2 \tag{6}$$

and taking into account equation (3), one obtains

$$X_{cn} = X_n - R_n\sqrt{1-(\mu_0/\mu_n)^2}, \tag{7}$$

while the coordinates of the next point on the trajectory are given by

$$X_{n+1} - X_{cn} = h + R_n\sqrt{1-(\mu_0/\mu)^2}, \qquad (9)$$

$$Y_{n+1} = Y_{cn}\sqrt{R_n^2 - [X_{n+1} - X_{cn}]^2}, \qquad (10)$$

where h is the step length.

It is possible to obtain, from equations (3)–(5) and (10), all of the points on the ray path with an accuracy determined by the magnitude of the step length h. The entire mathematical procedure was programmed in FORTRAN language and computations were performed on an IBM-360/44 computer in 200–2000 steps.

PLASMA REFRACTIVE INDEX

A general review of plasma refractive index theory has been given by ASCOLI-BARTOLI[2] and by ALPHER and WHITE.[3] Therefore, only minimal details needed for explanation of our work will be given here.

Considering a plasma as a mixture of electrons, neutral atoms and ionized particles, its refractivity may be expressed as

$$\mu = 1 - \tfrac{1}{2}(\omega_p/\omega)^2 + 2\pi\sum_i \alpha_i N_i, \qquad (11)$$

where $\omega = 2\pi\, c/\lambda$, c is the speed of light, λ is the wavelength, $\omega_p{}^2 = (N_e e^2/\varepsilon_0 m)$, N_e = electron concentration, ε_0 = permeability of the vacuum, e and m are the electronic charge and mass, respectively, N_i is the number density of atoms (or ions) in the ith quantum state, and α_i is the polarisability of this electronic state. Equation (11) has been further used for the computation of plasma refractive index for wavelengths between 4000 Å and 7000 Å. The wavelength dispersion of α_i has been neglected in these evaluations, as has also the contribution of excited species.[2]

For further examination of the deviation of the light rays, the plasma of a wall-stabilized hydrogen arc at atmospheric pressure has been chosen. There are several reasons for this choice: (a) a stable hydrogen arc can only be obtained in narrow-bore confinement tubes, i.e., with high density gradients; (b) there exist a number of very extensive spectroscopic measurements; (c) the plasma is close to the state of thermal equilibrium[5] and calculated composition data can be used, if necessary, with reasonable confidence. Since it is preferred to use a steady-state plasma with well known radial distribution of plasma parameters and high refractive-index gradients for the illustration of the effect of light refraction, the choice of the hydrogen arc is reasonable.

Two examples of the radial variation of the refraction index at λ = 5000 Å and 6500 Å are given in Fig. 2. These were evaluated for temperature profiles reported by WIESE *et al.*[5] and by STEINBERGER[6] for the arc burning in 3 mm and 2 mm dia tubes, respectively. The values are based upon the calculated composition of hydrogen at atmospheric pressure.[7] The data for mean electronic polarizabilities of hydrogen molecules and atoms are taken from ALLEN.[8] The influence of anomalous dispersion (the refractive indeces given in Fig. 2 were calculated at wavelengths in the vicinity of the hydrogen lines H_α and H_β) was neglected in these computations. This neglect is possible since the number density of electrons is

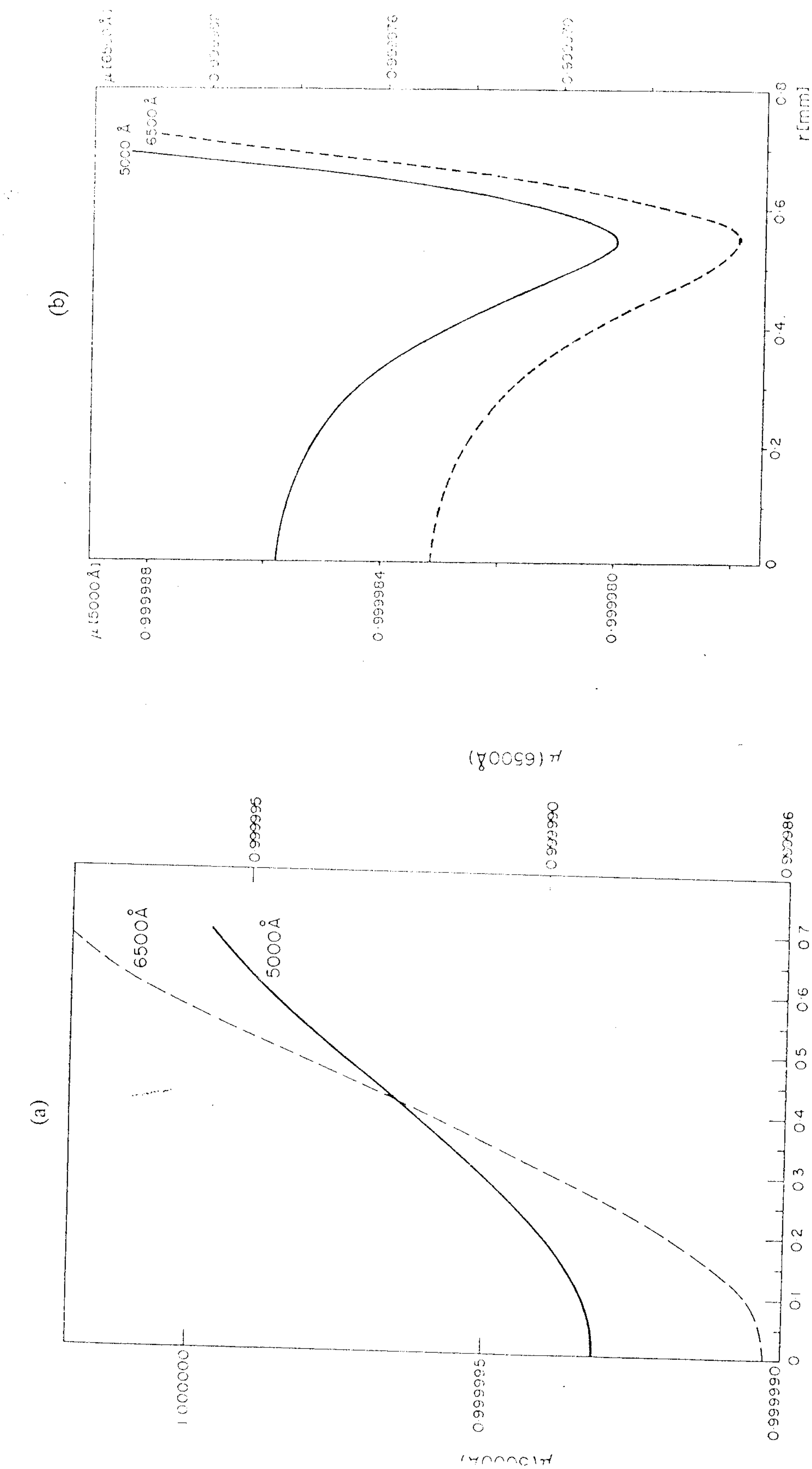

Fig. 2. The radial distribution of the refractive index in a hydrogen arc at 5000 Å (—) and at 6500 Å (---); (a) 3 mm *dia.* 40 A; (b) 2 mm *dia.* 150 A.

NUMERICAL RESULTS

From the radial distribution of the refractive index, Fig. 2, the trajectory of the radiation through the plasma has been computed using the procedure described previously and assuming that the plasma is optically thin. It is necessary to approximate the radial distribution of the refractive index with one of the curves described by equation (5). The distribution in Fig. 2b comprises two parts: from the minimum of the refractive index to the axis of the arc and from the minimum to the arc periphery.

The results for the ray paths through the plasma are given in Fig. 3, where the deviation

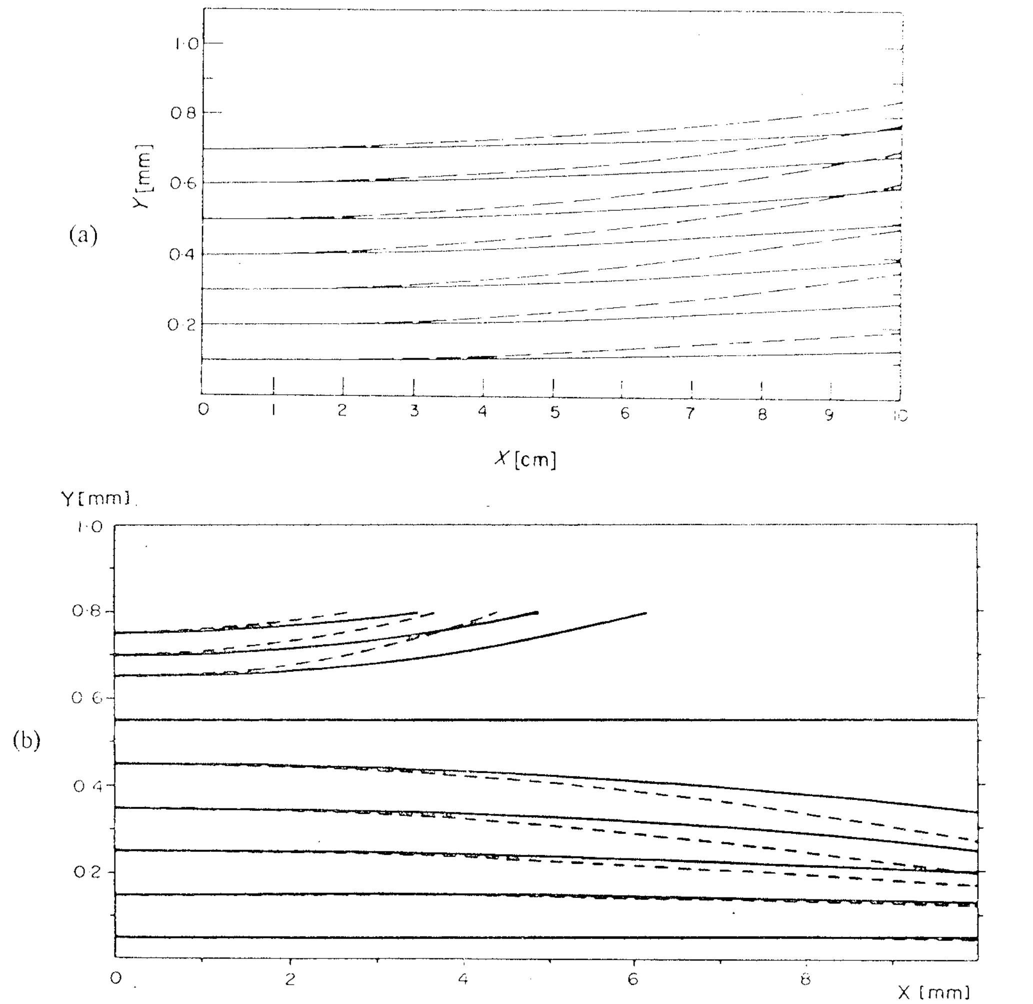

Fig. 3. Ray trajectories through the plasma at 5000 Å (—) and at 6500 Å (---); (a) 3 mm dia., 40 A;[7] (b) 2 mm dia., 150 A.[8]

of the light ray is given as a function of the plasma column length. Figure 3 gives the magnitude of deviation showing that, for a high-current arc,[6] end-on spectroscopic measurements at 6500 Å may be affected by the Schlieren effect even at a plasma length of a few centimeters. Obviously, the distortion decreases with the wavelength (refractive index gradients are mainly due to the refraction of electrons) and it is practically negligible below 3500 Å for the experimental condition treated in this paper. Therefore, a comparison of the radial distribution of the continuum radiation taken in the u.v. and in the visible regions of the spectrum should be made with caution since the latter may be distorted. This effect is

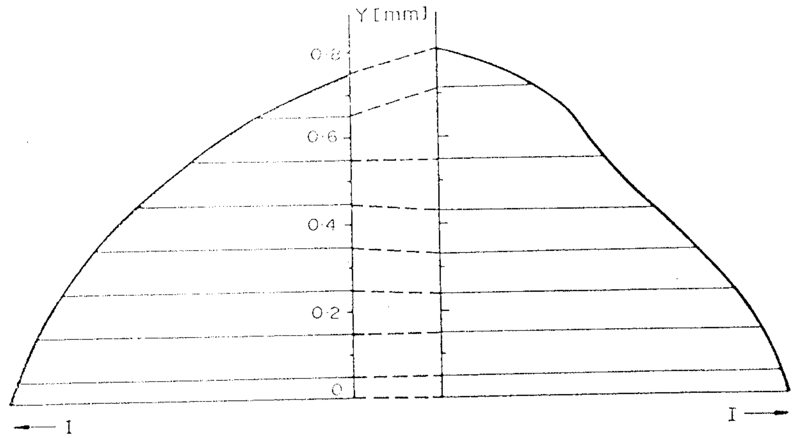

Fig. 4. Distortion of the radial light intensity distribution at 6500 Å by 3 cm of plasma in a 150 A arc.[8]

Finally, an attempt has been made to assess the magnitude of the deviations in the case of side-on plasma observations. It was found negligible for the experimental conditions treated in this paper. In a way, this result can be seen from Fig. 3, where no deviation is noted through the plasma length of few milimetres in the presence of gradients perpendicular to the ray path [which correspond to the case of largest bending of a ray, see equation (1)]. This situation does not arise in the radial direction in cylindrically-symetric plasmas, where the deviations are even smaller.

CONCLUSIONS

The results presented here show that refraction of the radiation along the axis of the plasma may introduce a systematic error in spectroscopic end-on measurements. The magnitude of this error depends upon the refractive index gradient and the plasma length.

The detailed mathematical procedure for the evaluation of this effect is given. Although it has been applied to the hydrogen arc, it may be used for any axisymmetric plasma.

Finally, it has been shown that bending of the side-on light ray in the plasma may be neglected. However, if measurements are taken further in the infrared region, the influence of the Schlieren effect ought to be examined carefully.

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