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METHOD FOR FAST CARRY-PROPAGATION FOR VLSI ADDERS

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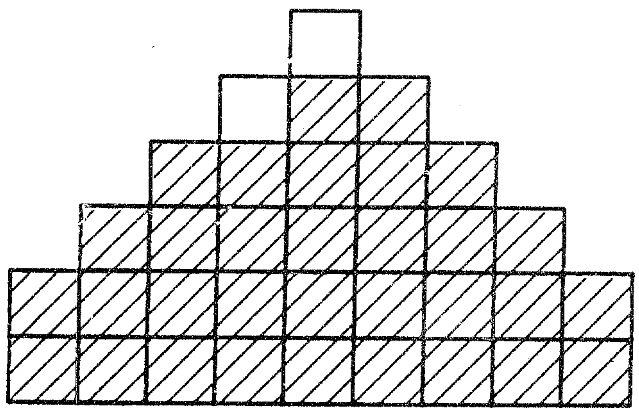

FIG. 1

A carry circuit is disclosed which is faster than conventional carry-skip techniques and yet requires about the same amount of circuitry, making it convenient for very-large-scale integration (VLSI) implementation. The improvement in speed is achieved by dividing the carry chain into groups of bits of various sizes. The sizes are chosen to maximize the speed of addition.

Let δ_{max} denote the maximum time required to add any two n-bit numbers. The technique gives

$$\delta_{max} = 2\sqrt{n+2} - 3.$$

In conventional carry-skip adders where the carry chain is divided into groups of equal size k, one has

$$\delta_{max} = \left(\frac{n}{k} - 2\right) + 2(k-1).$$

A comparison of the new scheme with the conventional one for a 32-bit adder is shown in Fig. 2.

Method

(i) Given n, the number of bits in the adder, define $\delta = 2\sqrt{n+2} - 3$ to be the largest integer $\leq 2\sqrt{n+2} - 3$. For i = 1, 2, ..., δ, define

$$y_i = \min\ 1 + i,\ \delta + 2 - i.$$

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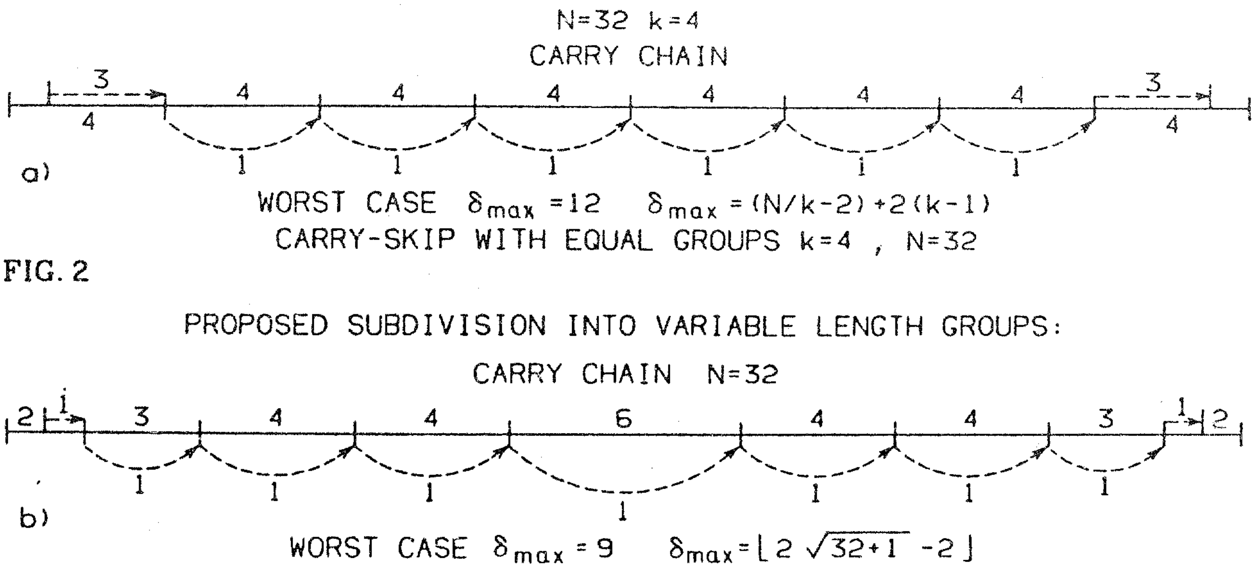

FIG. 2

Construct a histogram whose i-th column has height y_i, $i = 1,\ldots,\delta$. For example, the histogram corresponding to a 32 bit adder is shown in Fig. 1.

It is easy to show that the number of squares in the histogram is

$$\delta + \frac{1}{2}\delta + \frac{1}{4}\delta^2 + \frac{1}{8}(1-(-1)\delta) \geq n.$$

Thus, the histogram contains at least n squares.

(ii) Shade in squares of the histogram one row at a time until n squares are shaded. Moving left to right, let x_i denote the number of shaded squares in column i of the histogram, $i = 1,\ldots,\delta$. X_1, $x_2,\ldots,x_\delta$

give optimal group sizes for a division of the carry chain. For example, from Fig. 1 that $x_1 = 2$, $x_2 = 3$, $x_3 = x_4 = 4$, $x_5 = x_6 = 5$, $x_7 =$

4, $x_8 = 3$, $x_9 = 2$ gives optimal group sizes for a 32-bit adder.

<u>Proof that δ is the Longest Possible Path</u>

It is clear from the construction of the groups that

$$x_i \leq \min(i+1,\ 2+\delta-i)$$

Let P be a path starting in the i-th group and ending in the j-th group, i<j. Let L(P) denote the length of P. It will show that $L(P) \leq \delta$.

$$L(P) \leq x_i - 1 + x_j - 1 + j - i - 1$$

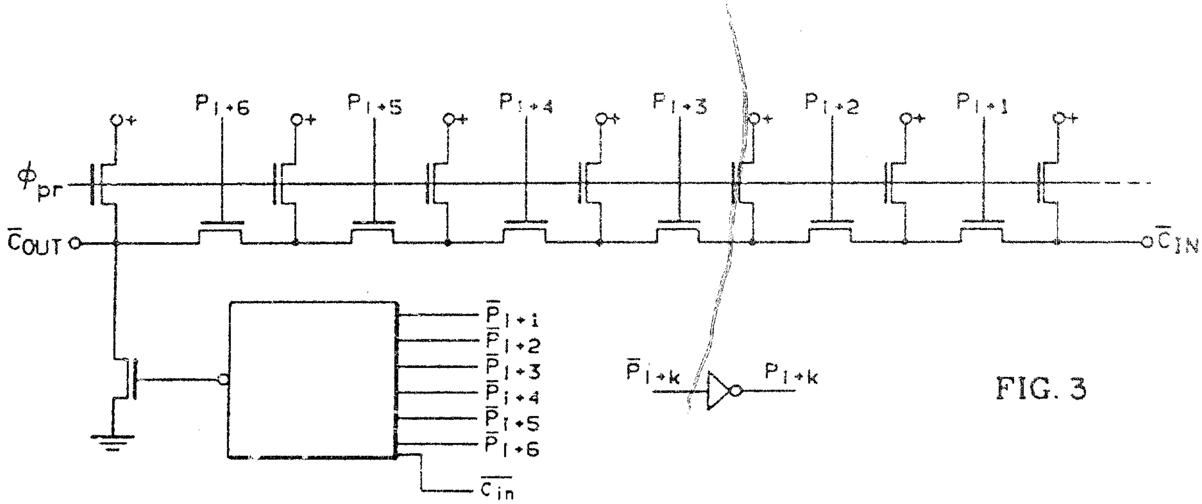

FIG. 3

$$\leq \min(i+1, 2+\delta-i) + \min(j+1, 2+\delta-j) + j-i-3$$

There are three cases to consider. First, assume that

$$i+1 \leq 2+\delta-i \text{ and } j+1 \leq 2+\delta-j.$$

These conditions imply that $2j-1 \leq \delta$. It follows that

$$L(P) \leq i+1+j+1+j-i-3 = 2j-1 \leq \delta.$$

Suppose now that

$$i+1 \leq 2+\delta-i \text{ and } j+1 \geq 2+\delta-j.$$

In this case, it is implied that

$$L(P) \leq i+1+2+\delta-j+j-i-3 = \delta.$$

Finally, assume that

$$i+1 \geq 2+\delta-i \text{ and } j+1 \geq 2+\delta-j.$$

Then $2i-1 \geq \delta$ and

$$L(P) \leq 2+\delta-i+2+\delta-j+j-i-3$$

$$= \delta-(2i-1)+\delta \leq \delta.$$

Proof that this Subdivision is the Best

It must be shown that there is no subdivision of 0,n for which the longest path is shorter than $\delta = \lceil (2\sqrt{n+2} - 3) \rceil$. Suppose 0,n is divided into k subgroups of lengths $x_1, x_2, \ldots, x_k$.

First, consider the case k is even. Then, k=2m is some integer m. Let L denote the length of the longest path for this subdivision. Then, L is greater than the longest of the paths starting in group i and ending in group k-i+1, i=1,2,...,m.

Also, $L \geq k=2m$. These equations are written as:

$$x_1 - 1 + x_k - 1 + k - 2 \leq L$$

$$x_2 - 1 + x_k - 1 + k - 4 \geq L$$

$$x_m - 1 + x_{m+1} - 1 + k - k \leq L$$

$$2m \leq L.$$

By adding these equations, using the fact that $\sum_{i=1}^{k} x_i = n$, the following is obtained.

$$n - 2m + mk - 2(1+...+m) + 2m \leq (m+1)L$$

This simplifies to

$$\frac{n + m^2 - m}{m+1} \leq L.$$

By differentiating the left side of this inequality with respect to m and setting the derivative to 0, the minimum of the left side over all values of m occurs at:

$$m+1 = \sqrt{n+2}$$

Substituting this

$$\frac{N + (\sqrt{n+2} - 1)(\sqrt{n+2} - 2)}{n+2} = 2\sqrt{n+2} - 3 \leq L$$

But since L is an integer it follows that

$$\lceil (2\sqrt{n+2} - 3) \rceil = \delta \leq L.$$

A similar analysis shows that $\delta \leq L$ for k = 2 m + 1 add. It follows that $\delta = \delta_{max}$ is optimal.

The above algorithm was implemented by a PASCAL program for n = 4 to 64. Implementation of the group subsection of the carry chain is shown in Fig. 3. A precharge technique is used because it assures better performance. In the case of the 32-bit adder, simulation results using the prameters for MOSFET technology show this scheme to be 20% faster than carry skip with the group length fixed at k = 4, yet the difference in complexity is negligible.